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THE PROBLEM OF DETERMINING THE EXCITATION FORCES IN VIBRATION TESTING

by G. de Vries

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THE PROBLEM OF DETERMINING THE EXCITATION FORCES IN VIBRATION TESTING

By G. de Vries

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THE PROBLEM OF DETERMINING THE EXCITATION FORCES IN VIBRATION TESTING /43

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Determining the excitation forces is at the same time the most important and the most difficult problem in vibration testing.

The reasons why the known theories in this field have not led to practical testing methods are discussed. The real behavior of a structure is compared with the theoretical hypotheses and the differences observed explain the reasons why the known methods are not always applicable. Following the demonstration that the appropriation, -- even when possible --, does not provide real advantages for measurement, it is suggested to reverse the problem by establishing the structure determination by means of a modification of the damping forces distribution for the separation of the modes.

Author

I. THE PROBLEM OF DETERMINING THE EXCITATION FORCES.

The problem of determining the excitation forces became important in the practical planning of vibration testing after the O.N.E.R.A. decided to follow the suggestion of Professor R. Mazet, Director of the Department of "Resistance of Structures" (1) and to base the calculations of stability in flight entirely on the results of vibration tests. In this manner it became possible to forego the difficult and uncertain calculations inherent to division of a structure with regard to its component masses and their rigidities.

Vibration testing which must replace calculations and static testing poses many problems, the most difficult of which proved to be that of division of the excitation forces for isolation of different vibrating modes, or the problem of "apportioning of the forces of excitation".

In practical planning, none of the proposed solutions leads to satisfactory results, if excitation is applied to a whole complicated structure such as occurring in an airplane. This can be explained by the dynamic behavior of structures which is different from that used in theoretical calculations.

Analysis of publications on the subject of apportioning reveals the following hypotheses and criteria:

1. Linearity of structural parameters (all authors).

/Numbers in the margin indicate pagination in the original foreign text.

2. Constancy of a structure as a function of time (all authors).

3. Basile hypothesis (3, 12).

4. Accessibility of all points of a structure for excitation and measurement (all authors).

5. Resonance phase criterion (all authors).

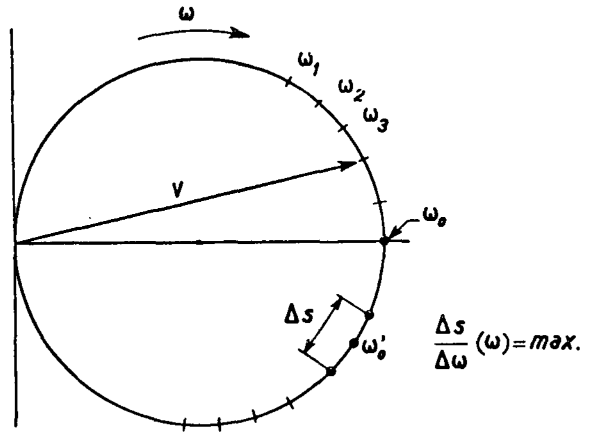

Fig. 1. Definition of a characteristic frequency according to the criterion of phase (ω_0) resonance and to that of Kennedy and Pancu (ω_0').

To ensure the generality of a procedure it is necessary that the test structure should verify, for all modes, that the incidence of eventual variation between the hypothesis and the results be small. As is proved by experience, a structure is never rigorously linear and the characteristic frequency of modes varies slightly with amplitude and displacement (14). This variation of characteristic frequency will affect all procedures based on the exact knowledge of resonance frequency (5, 7, 8). As is shown (14), these slight non-linearities cause an uncertainty with regard to characteristic frequency. This frequency will seem to be variable depending on the phase criterion of Kennedy and Pancu (15), the criterion used for its determination (Fig. 1). /44

Another source of uncertainty is the non-constancy of the structure as a function of time. Thus, one sometimes observes a considerable rotation of the response vector as a function of time for a constant excitation frequency and force of excitation. It is to be noted that the affix of the vector in all observed cases remains on the circle obtained for all ranges of frequencies The errors due to non-linearity are inversely proportional to the strength of damping in a very systematic manner. They cannot thus be eliminated by methods of averages or correlation (13).

The same phenomenon of non-linearity and evolution as a time function recurs in a more pronounced degree for damping forces. They especially affect those procedures which are endeavoring to establish a complex matrix of damping forces (6, 7, 8, 9). These procedures (with the exception of (7) which, according to the author of (8), do not always assure the convergence of iteration), necessitate intermediate calculations between two phases of testing and are, therefore, very sensitive to the variation of structural parameters as functions of time.

The Basile hypothesis is retained only by one author (3) as the working basis for an experimental process. Moreover, it provokes a remark with regard to the very definition of excitation force determination.

In fact, if the matrix of damping forces is diagonal (Basile's hypothesis), excitation forces pertaining to a mode excite only this mode. This is true not only for its resonance, but also for all frequencies of excitation.

Therefore, the mode is isolated, and the whole structure behaves as a simple system with one degree of freedom. This permits a utilization of the known relationships "force-response" of such a system in obtaining -- by experimental procedures -- the generalized values.

This particular method of isolating one mode is more general than that of "apportioning". For the latter, nothing is required but the response of the structure -- at resonance -- identical to the response of the structure without damping. It may be asked -- this will be discussed later -- whether the object of the study should not rather be that of isolating the modes obtained by adaption of a single shell structure usable for the evaluation of characteristic values: a structure with a diagonal matrix of dampers.

Let us consider at first, two hypotheses underlying all work concerning the study of apportioning; it will be shown that these hypotheses are similar with regard to conditions of vibration testing.

With n available excitors it is possible to eliminate (n-1) modes (4). This statement is, however, of very little practical value, because the modes can be eliminated only for one certain distribution (a priori unknown) of excitation forces, the study of which would necessitate a displacement of the excitors and would be impossible to achieve in the event that certain elements of the structure should be inaccessible. This occurs constantly in the case of airplanes and engines with their carburetors, propellers, their landing equipment, etc.

It is necessary to examine whether the phase criterion, which has already shown reservations for non-linearities, is at least valid for a perfectly linear structure. For this examination we shall consider the signs of a number m of censors for p modes, assuming that only these modes can measurably contribute to response $\vec{\nu}_m$ at point m.

At each point, the response is the vectorial sum of the partial responses of p modes:

$$\vec{\nu}_m = \sum_1^p \alpha_{mp} \cdot \vec{\nu}_p = \sum_1^p \alpha'_{mp} \cdot \mathfrak{R}(\vec{\nu}_p) + \sum_1^p \alpha''_{mp} \cdot J(\vec{\nu}_p). \tag{1}$$

The phases resonance condition for m points requires:

$$\sum_1^p \alpha''_{mp} \cdot J(\vec{\nu}_p) = 0 \tag{2}$$

This results in a system of equations:

$$\begin{array}{l} \alpha_{11} \cdot J_1 + \alpha_{12} J_2 + \ldots = 0 \\ \alpha_{21} \; J_1 + \alpha_{22} J_2 + \ldots = 0. \\ \ldots\ldots\ldots\ldots\ldots\ldots = 0. \end{array} \tag{3}$$

If $p > m$, this system has solutions other than $\vec{\nu}_p = 0$; $p \neq i$, ν_i representing the mode to be found.

If the phase criterion is realized for m points of the structure, this may indicate that the mode is excited at its proper frequency; this, however, may also represent a superposition of several modes. The phase criterion is, therefore, not general if the number of measurement points is limited.

The lack of success of experimental study can therefore be explained by the fact that thė hypotheses underlying the proposed procedures are not always verified and, secondly, by the limitation of the number of excitors and censors in the apparatus for vibration testing.

The fact is that the apportioning of excitation forces does not furnish mode isolation in the presence of non-diagonal terms of damping. The eventual complication of the test material and procedure by an increase in the number of excitors and censors is entirely unjustified. As a matter of fact a deformation can be obtained with a sufficiently great precision by vectorial decomposition (2, 16) and the measurement of the generalized mass will be neither facilitated nor rendered more precise by the apportioning. On the other hand, mode isolation, which we are about to investigate, may be advantageous in this respect.

II. STUDY OF CONDITIONS NECESSARY FOR MODE ISOLATION.

The most general case is presented by examination and study of mode isolation for a structure whose parameters are not strictly constant and do not conform to the Basile hypothesis. Under these conditions, the approach toward the solution is possible only by taking into consideration certain peculiarities of the structure to be tested.

II.1 Statement of the Problem. /45

As the first step we shall consider a slightly non-linear structure for which a linearization is still a fair approximation and which can be described by a system of simultaneous equations:

$$\left\{ -\omega^2 \left[\diagdown\mu\diagdown \right] + \left[\diagdown\varphi\diagdown \right] + j\omega \left[\beta \right] \right\} \left\{ q \right\} = \left| F \right|. \tag{4}$$

Let us assume that ϕ, β and ω are functions of amplitude, but are of small variability. We are in search of parameters of another system of linear equations:

$$\left\{ -\omega^2 \left[\diagdown\mu\diagdown \right] + \left[\diagdown\varphi\diagdown \right] + j\omega \left[\diagdown\beta\diagdown \right] \right\} \left\{ q \right\} = \left| F \right| \tag{5}$$

such that for a mode, the respective values μ, ϕ, ω and q of the two equations approach, as closely as possible, a given amplitude. The response of the mode according to (5) is known. For a proper excitation (which in this case conserves its sign), the response in velocity at each point of the structure is expressed by multiplying the constant k by the response of a simple model as in Fig. 2.

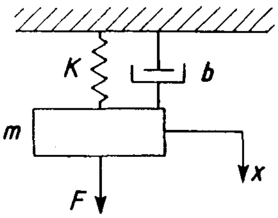

Fig. 2. Model for finding the coefficients of equation $m\ddot{\chi} + b\dot{\chi} + K.\chi = F$ at the start of dynamic response.

$$v = k \cdot \frac{F}{jm\omega + \frac{K}{j\omega} + b}. \tag{6}$$

We shall take this function as an example because it permits evaluation of generalized values if one knows ν for several frequencies of excitation.

We must operate a structure -- conforming to (4) -- in such a manner as to obtain a response of the form (6).

If this result is obtained, the effect of non-linearity can be eliminated by known methods (14) keeping this displacement modulus constant during the whole range.

II.2 Adaptation of a Structure to a Mathematical Matrix.

The essential difference between equations (4) and (5) consists in secondary parameters: the redistribution of damping forces. If one obtains a correction of this redistribution -- which is possible theoretically -- the test structure will conform to the Basile hypothesis and the modes can be treated according to (6). This correction can be realized by increasing the number n of dampers (electrodynamic, for example) of the forces capable of damping regulation.

In fact, the non-diagonality of the matrix is due to a defective rearrangement of damping forces. This may be corrected by the artificial addition of judicially arranged damping forces (Fig. 3). In this manner the global damping of all modes is increased. This does not always represent an advantage, but, in general, the effectiveness of electrodynamic dampers remains relatively small, if it is not desirable to augment the increase of mass by that of additional spools (18).

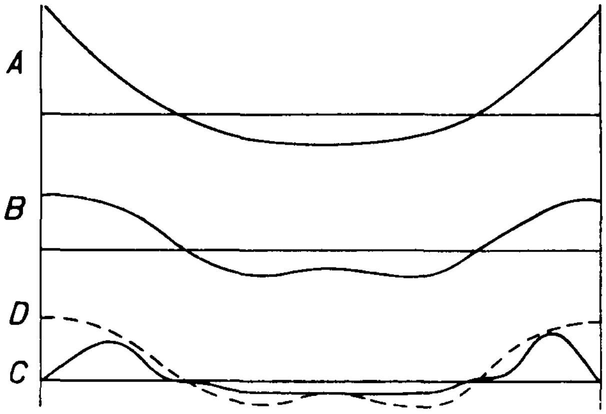

Fig. 3. Mode of a beam subjected to damping forces not satisfying the Basile hypothesis.

A. Deformed.
B. Redistribution of massive forces.
C. True redistribution of damping forces.
D. Redistribution of damping forces satisfying the Basile hypothesis.
D-C. Damping defect.

On the other hand, with these dampers supplementing the excitors proper, the experimental scope of possiblities is increased.

It seems more useful to employ the excitors which perform a two-fold function: that of a generator of excitation forces and that of a damper. In addition this permits an introduction of positive and negative damping forces.

II.3 Decomposition of the Excitation Force.

If damping is produced by force generators, the total force generated by excitors is decomposed into an excitation force proper, which is a function of the frequency generator signal, and into a positive or negative damping force which is a function of the response of the structure at the location of the censor (Fig. 4).

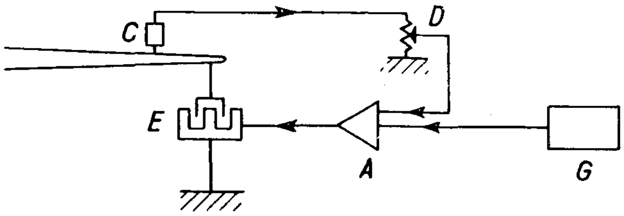

Fig. 4. Schema of reaction for producing damping forces by means of a principal excitor.

C. Velocity censor
D. Damping measurement
E. Excitor
A. Amplifier
G. Frequency generator

Applying this to the model in Fig. 3, one obtains: /46

$$\left(jm\omega + \frac{K}{j\omega} + b\right) \cdot v = F + \varepsilon v. \tag{7}$$

If modulus $|v|$ is kept constant, the sum on the right side of the equation must also remain constant for phase resonance when $jm\omega + \frac{K}{j\omega} = 0$. Apparently the reinjection does not produce any change.

Combining b and ε one obtains:

$$\left(jm\omega + \frac{K}{j\omega} + b - \varepsilon\right) \cdot v = F. \tag{8}$$

One sees, however, that the characteristics of the model are changed and that the sweep of frequency gives different responses outside of the resonance with regard to the distribution of parameter ω on the circle.

Applying the reinjection on the real structure described by (4),

$$\left\{ -\omega^2 \begin{bmatrix} \diagdown \\ \mu \\ \diagdown \end{bmatrix} + \begin{bmatrix} \diagdown \\ \varphi \\ \diagdown \end{bmatrix} + j\omega \begin{bmatrix} \beta \end{bmatrix} \right\} \left\{ q \right\} = \left| F \right| \tag{9}$$

if S_{ij} and S_{ih} are displacements of point i for modes j and h at the measurement point, and S_{kj} and S_{kh} are the displacements of the point of excitation k (or vice-versa), one obtains:

$$\left\{ -\omega^2 \begin{bmatrix} \diagdown \\ \mu \\ \diagdown \end{bmatrix} + \begin{bmatrix} \diagdown \\ \varphi \\ \diagdown \end{bmatrix} + j\omega \begin{bmatrix} \beta \end{bmatrix} \right\} \cdot \left\{ q \right\} = \left| F \right| + j\omega \begin{bmatrix} \beta' \end{bmatrix} \cdot \left\{ q \right\} \tag{10}$$

$$\begin{bmatrix} \beta' \end{bmatrix} = \begin{bmatrix} S_{ij} \end{bmatrix} \cdot \begin{bmatrix} \diagdown \\ \varepsilon_i \\ \diagdown \end{bmatrix} \cdot \begin{bmatrix} S_{kh} \end{bmatrix}$$

$$\left\{ -\omega^2 \cdot \begin{bmatrix} \diagdown \\ \mu \\ \diagdown \end{bmatrix} + \begin{bmatrix} \diagdown \\ \varphi \\ \diagdown \end{bmatrix} + j\omega \begin{bmatrix} \beta - \beta' \end{bmatrix} \right\} \cdot \left\{ q \right\} = \left| F \right| . \tag{11}$$

Equation (11) shows that two theoretical operations are necessary for isolating a mode:

a) it is necessary to eliminate by (β') non-diagonal terms of (β). This is the "adaptation" of the structure;

b) it is necessary to choose a generalized force |F| such that

$$j\omega \begin{bmatrix} \diagdown \\ \beta'' \\ \diagdown \end{bmatrix} \cdot \left\{ q \right\} = \left| F \right| .$$

In practice it is not possible to separate these two operations and this is really not necessary because operation a) may contain the condition

$$j\omega \cdot \begin{bmatrix} \diagdown \\ \beta_j'' \\ \diagdown \end{bmatrix} \cdot \left| q_j \right| = \left| \overline{F}_j \right|$$

where $|\vec{F}_j|$ represents the existing generalized forces. This reverses the problem: instead of trying to find forces pertaining to a mode, one tries to find the proper damping of a group of previously selected excitation forces.

This procedure becomes more valid, as more difficulties of apportioning are encountered, in general, after the first approach operation of ideal excitation by means of several trials with different reapportioning of forces. The excitation forces obtained in this manner already constitute a good approximation of the desired generalized force. For improving its value, the elimination of non-diagonal terms of |β| is of an equal, if not greater, importance than the problem consisting of making identical the columns of excitation and damping forces.

Nevertheless, even in this simplified aspect, the problem is still too complicated for one to have hope of finding a practical procedure for testing. It is also well to profit by certain characteristics of the structure to be tested.

II.4 Characteristics of a Real Structure.

A homogeneous structure such as represented by a beam or a beam assembly, rarely presents any difficulties for mode separation. This explains why the refined laboratory vibration testing of such structures as metal plates, beams, clay models, etc....., is easy. This however, may lead an investigator to an erroneous conception with regard to the validity of measurement procedure.

In fact, a complicated structure such as that of an airplane, for example, also represents a more or less homogeneous part, but it is carrying suspended or articulated elements such as rudders, landing gear, propellers, carburetors, equipments, etc...

The homogeneous part has relatively low values of damping concentrated at the assembly points. The greatest part of measured global damping arises from the motions of accessories relative to that of the structure. Therefore, it becomes necessary to find the origin of ascertained non-linearities and inaccuracies.

On the basis of such a conception it is possible to establish a certain order in the research with regard to mode isolation. For evaluation, it is useful to divide the structure into subassemblies in such a manner that this division could not prevent discussion with regard to the modes of the whole structure, but could lead to a convenient reapportionment of excitation forces and supplementary damping.

Let us attempt to examine the useful arrangement of the excitors, while utilizing the gathered theoretical information and facts observed during testing.

1. For the homogeneous part of construction -- which we shall call the structure -- the number of necessary excitors is apparently very small. It does not seem useful to prognosticate the supplementary damping forces, if the hypothesis of the concentration of structural damping forces is exact.

The vibration tests confirm this opinion because the modes concerning, essentially, the deformation of the structure are always easy to isolate (for example, the modes of bending).

2. The greatest difficulty always occurs during separation of two modes of neighboring characteristic frequencies. The frequencies proper of a structure are, in general, sufficiently different and the possibility of eventual proximity of the two frequencies can be explained by neighboring resonances of the structure and of the subassembly (or else those of the two subassemblies of the construction; wing and empennage, for example).

Under these conditions the theory permits only two possibilities compatible with the fact of two resonance frequencies approaching one another: either the coupling existing between the subassemblies is weak, or the vibration energy of one subassembly is negligible with regard to that of the other.

The first of these cases is more frequently encountered. The rudders, /47 considered as subassemblies with respect to structure, are good representatives of this case.

The schema of Fig. 5 shall be used for making certain useful conclusions for the practical realization of mode separation. The model represents a section of wing in bending, of rigidity K_1. In parallel with this rigidity there is a damper b_1. It is assumed that structural damping $2\alpha_1 = \frac{b_1\omega_1}{K_1}$ is weak, for example, 5 ^{o}/oo. The rudder inertia is weak in comparison with the mass of the structure and,

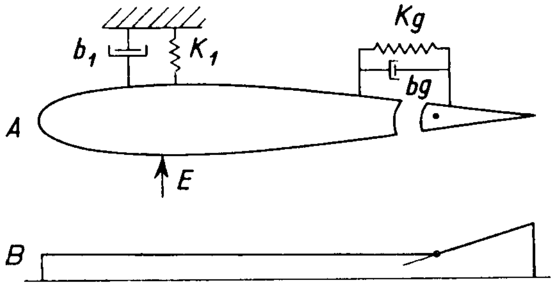

Fig. 5. Model for the bending mode-wing, torque aileron.

A. Model
B. Deformed

since we assume that characteristic frequencies of the structure and of the rudder are similar, it follows that $K_1 >> Kg$.

The same hypothesis is not valid for damping forces: on the contrary, the force of friction in the operating mechanism is of the value b_g which is of considerable magnitude, so that

$$2\alpha_g = \frac{b_g\omega_g}{K_g} \gg 2\alpha_1.$$

In this case the Basile hypothesis is not valid, but the damping b_g is localized. Let us observe that there exists a means for measuring the order of magnitude of α_g, which is the measure of relative phases (Fig. 6).

The obtained conclusions are:

a) Each articulated or suspended element must be provided with an excitor permitting simultaneous introduction of damping which, in general, is negative.

b) Coupling between these subassemblies being weak, the correction of damping may proceed successively for each of the elements, because the reaction of a modification transferred from one subassembly to the others and to the structure remains equally weak.

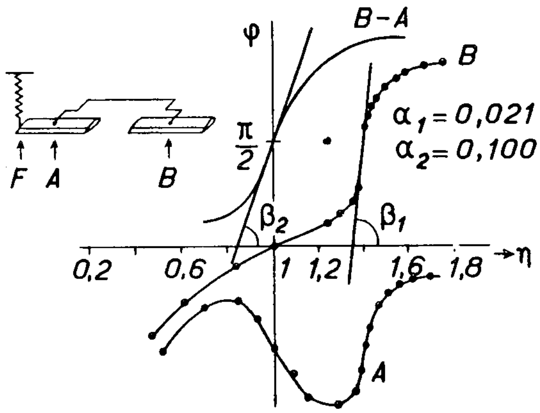

Fig. 6. Determination of characteristic frequencies of subassemblies and their damping by the combined methods of measurements of angles of relative phases and the method of slope $d\phi/d\eta$ to resonance according to (2).

II.5 Criterion of Study of the Apportionment.

Phase criterion may serve only as a first step of iteration. In the following steps, it must be assured that the mode obey the mathematical matrix used for application of the measurement results.

The non-linearities complicate the operations by the fact that the modulus of the global amplitude of the structure must remain constant during the total range of frequencies (this is necessary for the measurement of the normal frequencies and that of the generalized masses).

The method of force quadratures (14) seems to be most suitable among all possible procedures. When it is applied in the study of structure adaption to a given excitation, this method consists of three phases:

A. Excitation at presumed resonance frequency by group of forces already adapted to the mode; measurement of the frequency of responses at a certain number of points characterizing the deformation and especially the response of the subassembly for which damping is to be corrected.

B. Addition of the quadrature of the force to $|F|$ in such a manner that the excitation force becomes $F(1+j\lambda)$, where $\lambda<<1$; determining the phase resonance relative to F by varying the frequency of excitation; measurement of new frequency ω_1; measurement of response.

C. Repetition of operations B for F $(1-j\lambda)$.

The criterion is double: it is necessary that $|\Delta\omega_1| = |\omega_o - \omega_1|$ be equal to $|\Delta\omega_2| = |\omega_o - \omega_2|$ and, on the other hand, it is necessary that the modulus of amplitudes be constant during these three operations. In all cases, excluding exceptional ones, which remain to be discussed, the operations can be conducted separately for the structure and the subassemblies in a practical order.

The criterion is valid for linear and slightly non-linear constructions; the complication (three measurements replacing a single one) is the price of its being universal.

A more rapid criterion of application is given by the relaxation. In fact, the action of the damping forces continues after the abatement of excitation forces. If the non-diagonal terms of damping are annulled by reinjection, and if on the other hand excitation is accommodated, the curve of the relaxation must be the same for all points of the object. Therefore, it is sufficient to verify whether the "rights" of Lissajous which are obtained on the oscilloscope for the signals of the censors remain the "rights" of the curve of the relaxation. Control is more rapid and is made easier by diminishing global damping of the mode under consideration while increasing the disturbance mode by the choice of censors used for reinjection.

Even in the presence of non-linearities, the phase condition during /48 the progress of the relaxation remains valid at least for the initial periods.

III. SPECIAL CASES

The presented method permits a treatment of slightly non-linear structures with certain reapportioning of damping forces if they remain smaller than the elastic reaction forces. Another restriction is that the subassemblies be accessible.

If these conditions cannot be realized, it becomes necessary to consider the consequences resulting from the conduction of tests and the results of measurement.

III.1 Element with Highly Non-linear Characteristics.

The classical case encountered in testing highly non-linear elements is that of a rudder with the steerage of an activating apparatus. This case is one of dry friction damping and, at the same time, of rigidities. The schematic representation of this case is shown in Fig. 7.

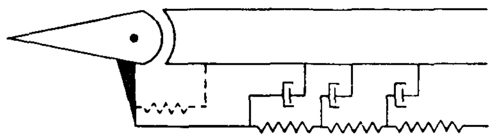

Fig. 7. Schema with several springs and dampers representing the steering mechanism of a rudder. Dotted lines represent rigidity of mechanism for vibration testing.

For a sufficiently large amplitude of rotation the steerage assembly is displaced together with the rudder. Following known methods of comparing the energies, it is possible to compare friction forces with their equivalents of viscous damping and express them with the use of a coefficient which nevertheless is a function of amplitude.

It is not always possible to attain the necessary amplitudes to free the steerage from friction against its encasement because rudder amplitudes are tied up -- by deformation mode -- with the amplitudes of the structure. For a mode of a high ratio $\frac{\text{amplitudes of structure}}{\text{amplitudes of rudder}}$, the rudder amplitudes are limited by permissible amplitudes of structure. In this case there may occur a partial encasement of steerage apparatus and the characteristic frequency of the rudder may change by a considerable degree.

There is a chance that this fact may not be perceived during the test, because in the vicinity of a given amplitude the test structure behaves as a new structure and its behavior is perfectly linear. It is evident that in this case it is useless to try to compensate the damping because it would then be necessary to reach the locations of friction, which are rarely accessible.

A structure of these characteristics cannot be made amenable to any mathematical matrix, which could be used for determination of true and deformed values. Therefore the structure must be modified before testing either by eliminating the sources of friction or by replacing the non-linear element with an equivalent element of linear characteristics (Fig. 7).

Another possibility consists of modifying the structure and then taking into consideration these modifications in subsequent calculations. Also, the ratio of $\frac{\text{structure amplitude}}{\text{rudder amplitude}}$ can be modified by adding certain masses to the rudder to increase the value of coupling.

In this case, the modifications which were made to render the structure accessible to measurement of modes, concern not only the damping, but also the masses and rigidities. The results of measurement must, therefore, be corrected for these modifications. This is not necessary if only the division of damping forces is affected by these corrections.

III.2 Non-accessible Subassemblies.

A non-accessible subassembly produces a hidden parameter effect as discussed in (17). The fact that deformation of such elements is not measurable, does not hinder stability calculations of airplanes and engines. This is so because it is of no importance in calculation of the aerodynamic forces. It remains to determine the repercussion of unsuitable excitation due to inaccessibility of certain subassemblies.

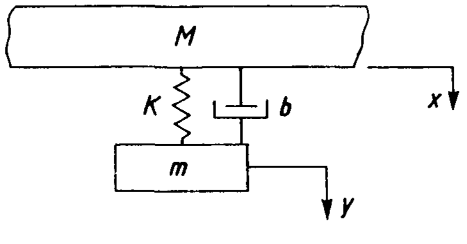

Fig. 8. Model for discussion of the effect produced by a hidden parameter. M represents the structure, X -- the displacement at the point of contact for a mode.

For a model shown in Fig. 8, the relative mass displacements are:

$$x - y = \frac{-m\omega^2}{-m\omega^2 + K + jb\omega} \cdot x \tag{12}$$

and the force at the point of contact is:

$$F = -K \cdot \frac{\eta^2 \cdot (1 + 2j\alpha\eta)}{1 - \eta^2 + 2j\alpha\eta} \cdot x \tag{13}$$

with

$$\omega^* = \sqrt{\frac{K}{m}}; \ \eta = \frac{\omega}{\omega^*}; \ 2\alpha = \frac{b}{m\omega^*} = \frac{b\omega^*}{K}$$

There can occur two principal cases:

<u>The first case</u>: Mass m is large; K is also large. Due to coupling, the characteristic frequency ω^* of the uncoupled subassembly is necessarily quite different from the characteristic frequency ω_S of the mode.

With excitation up to frequency ω the real part of force F of contact, for weak values of $\alpha \ll 1$ becomes:

$$F' \approx -\frac{K\eta^2}{1 - \eta^2} \cdot x \tag{14}$$

It is the same as would have been obtained for a proper excitation; the measured mass is then -- in a good approximation -- the same as that of the assembly. The imaginary part of the contact force becomes:

$$F'' \approx - K \cdot 2\alpha \cdot \frac{\eta^3}{1 - \eta^2} \cdot x \qquad (15)$$

Comparing the imaginary part of the contact force with the imaginary force of a damper which replaces the assembly K,m,b, one arrives at the following conclusions: in interval $0.5 < \eta < 1.5$, excluding the zone $0.9 < \eta < 1.1$, where the secondary terms are no longer negligible, the error of the imaginary part with respect to the equivalent of a damper, does not exceed 10% in the range of 1% about the resonance frequency. Knowing that the imaginary force at the point of contact is but a small fraction of all damping forces, its assimilation with these forces will introduce a negligible error in apportioning which, therefore, can be obtained by placing an excitor with reinjection at the point of contact. /49

The second case: (more interesting for testing) is that of two neighboring frequencies $\eta \simeq 1$. In this case it is again assumed that there is weak coupling between the structure and the inaccessible subassembly. In this case a general discussion is no longer possible. In fact, the force at the point of contact is the subject of rapid evolution as a function of the frequency of excitation, and the response curve is severely disturbed.

If it is not possible to locate the responsible subassembly and to intervene by supplementary damping, the only possibility is to increase the range of the frequency scale and to eliminate the values of the perturbed zone in the response.

IV. CONCLUSION

Due to non-linearities and inconstancies of structures to be measured, such as those of airplanes and engines, the known methods of determination of correct forces are unable to lead to satisfactory results. Knowing that advantages obtained by apportioning the forces of excitation are in all cases very small, a further complication of test methods and material is not justified.

On the contrary, conforming the structure to mathematical matrices has the advantages of being general and equally valid for stationary and transitional regimens. It offers criteria of rapid application and permits selective variation of structural mode damping.

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